ARCHER Training Courses

Naïve Bayes Classification

- Please feel free to ask questions as we go along

Reusing this material

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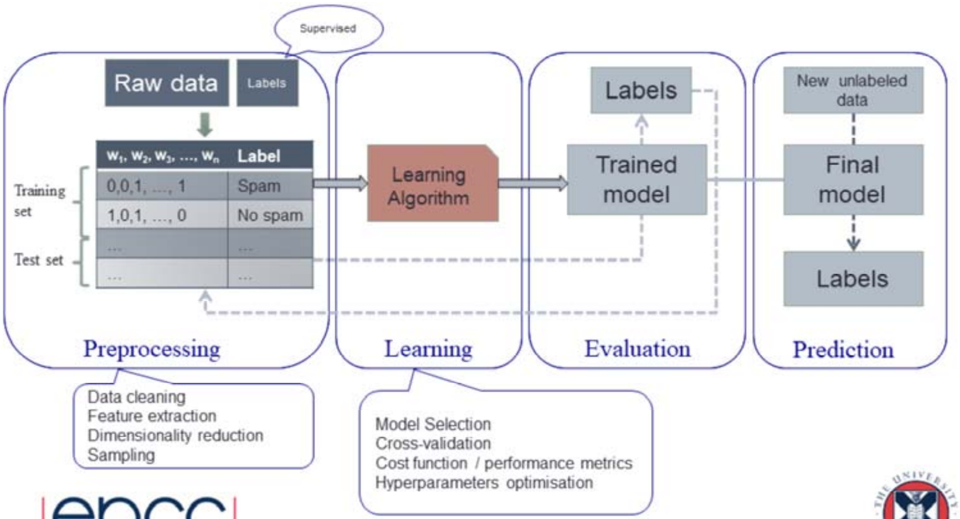
Supervised Machine Learning Classification



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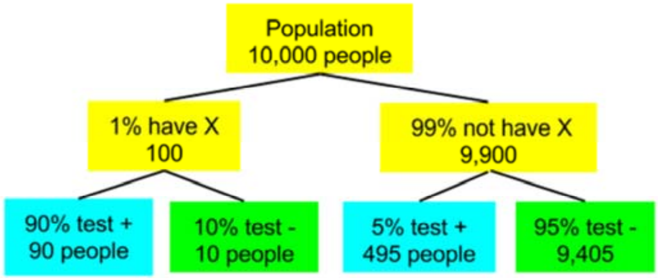
Revision and question

- Test available for condition X
- Sensitivity is 90%
  - For 90% of people with condition X the test will be positive
- Specificity is 95%
  - For 95% of people without condition X the test will be negative

Q. If I take the test and the result is positive what is the probability that I have condition X?

Answer

- It depends on the rate at which condition X occurs in the population



- Thus 90+495 = 585 people test positive, of these 90 have condition X. This is 15.4%

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Bayes' Law

- Definitions:
  - $p(x)$: probability of event x
  - $p(x, y)$: probability of event x and event y (independent or otherwise)
  - $p(x|y)$: probability of event x given event y

$$p(x, y) = p(x|y)p(y) = p(y|x)p(x)$$

- Bayes' Law:

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)} = \frac{p(x|y)p(y)}{p(x|y)p(y) + p(x|\neg y)p(\neg y)}$$

- Bayes' Law shows importance of overall event probability
- Allows to measure $p(x|y)$ and calculate $p(y|x)$

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Bayes' Law applied to example

- Bayes' Law

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)} = \frac{p(x|y)p(y)}{p(x|y)p(y) + p(x|\neg y)p(\neg y)}$$

- Applied to example:

$$p(X|+) = \frac{p(+|X)p(X)}{p(+|X)p(X) + p(+|\neg X)p(\neg X)}$$

Test sensitivity

Probability of X in whole population

$$p(X|+) = \frac{0.90 \times 0.01}{0.90 \times 0.01 + 0.05 \times 0.99} = 0.154$$

1 – Probability of X in whole population

1 - Test specificity

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Probabilistic Classification Model

- We wish to build (train) a statistical model that we can use to classify instances of observed data
- C is set of classes
  - e.g. $C = \{\text{Spam}, \text{NotSpam}\}$
  - e.g. $C = \{\text{Setosa}, \text{Versicolor}, \text{Virginica}\}$

Setosa Versicolor Virginica

- Observed data vector x for n features

$$x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}$$

- Want a model that allows us to calculate $p(c_i|x)$ for all $c_i \in C$
- Then simply choose the class that gives the highest value

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Applying Bayes' Th. and being naive

- How do we measure $p(c_i|x)$?
  - Bayes' Theorem!

$$p(c_i|x) = \frac{p(c_i)p(x|c_i)}{p(x)}$$

- $p(x)$ is same of all c_i so we can ignore that:

$$p(c_i|x) \propto p(c_i)p(x|c_i)$$

- Now we can be naïve and assume independence between all features:

$$p(c_i|x) \propto p(c_i)p(x_1|c_i)p(x_2|c_i)...p(x_n|c_i)$$

SPAM classification

- Wish to classify emails as SPAM or not SPAM
- Data for each email:
  - Sender
  - Subject
  - Message contents
  - Lots of other metadata :Sender's IP, time, …
- Simple method:
  - Classify on presence or absence of keywords
    - Viagra, HPC, purchase, cash etc.
- Is K-NN suitable?
  - No.
    - With 1000s of words there are too many dimensions
    - Dimensions not weighted to relevance

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Naïve Bayes Classifier and SPAM

- Recall:

$$p(c_i|x) \propto p(c_i)p(x_1|c_i)p(x_2|c_i)...p(x_n|c_i)$$

$$p(c_i) = \frac{\text{number of emails of class } c_i}{\text{number of emails}}$$

- Assume the features are the top 10,000 words

$$x_j = \begin{cases} 1, & \text{if the email contains word } w_j \\ 0, & \text{otherwise} \end{cases}$$

$$p(x_j|c_i) = x_j p(w_j|c_i) + (1 - x_j)(1 - p(w_j|c_i))$$

$$p(w_j|c_i) = \frac{\text{number of emails of class } c_i \text{ that contain word } w_j}{\text{number of emails of class } c_i}$$

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SPAM example

- Training set 500 spam, 800 non-spam
- “viagra” occurs in 234/500 spam, 10/800 non-spam
- “epcc” occurs in 100/500 spam, 300/800 non-spam
- “Bayes” occurs in 2/500 spam, 56/800 non-spam

$$p(\mathrm{Spam}|\{\mathrm{viagra}=1,\mathrm{epcc}=0,\mathrm{Bayes}=0\})$$

$$\propto p(\mathrm{Spam})p(\mathrm{viagra}|\mathrm{Spam})(1-p(\mathrm{epcc}|\mathrm{Spam}))(1-p(\mathrm{Bayes}|\mathrm{Spam}))$$

$$=\frac{500}{500+800}\cdot\frac{234}{500}\cdot(1-\frac{100}{500})\cdot(1-\frac{2}{500})=0.143$$

The email is classified as spam

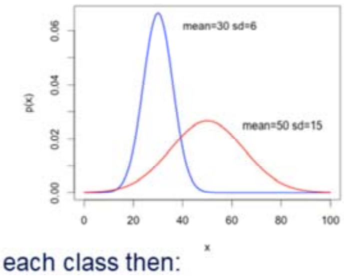
$$p(\mathrm{NonSpam}|\{\mathrm{viagra}=1,\mathrm{epcc}=0,\mathrm{Bayes}=0\})$$

$$\propto p(\mathrm{NonSpam})p(\mathrm{viagra}|\mathrm{NonSpam})(1-p(\mathrm{epcc}|\mathrm{NonSpam}))(1-p(\mathrm{Bayes}|\mathrm{NonSpam}))$$

$$=\frac{800}{500+800}\cdot\frac{10}{800}\cdot(1-\frac{300}{800})\cdot(1-\frac{56}{800})=0.0045$$

Gaussian Naïve Bayes

If we have continuous values rather than discrete then simply model with appropriate distribution



- Gaussian (normal) distribution
  - Mean (μ) and variance (σ^2)
  - σ is called standard deviation
  - 95% of data lines within 1.96 x standard deviations of the mean
- For each feature calculate mean and variance for each class then:

$$p(x_j|c_i) = \frac{1}{\sigma_{i,j}\sqrt{2\pi}} e^{-\frac{(x_j-\mu_{i,j})^2}{2\sigma_{i,j}^2}}$$

- Mean and variance can be calculated in a single pass through the data.

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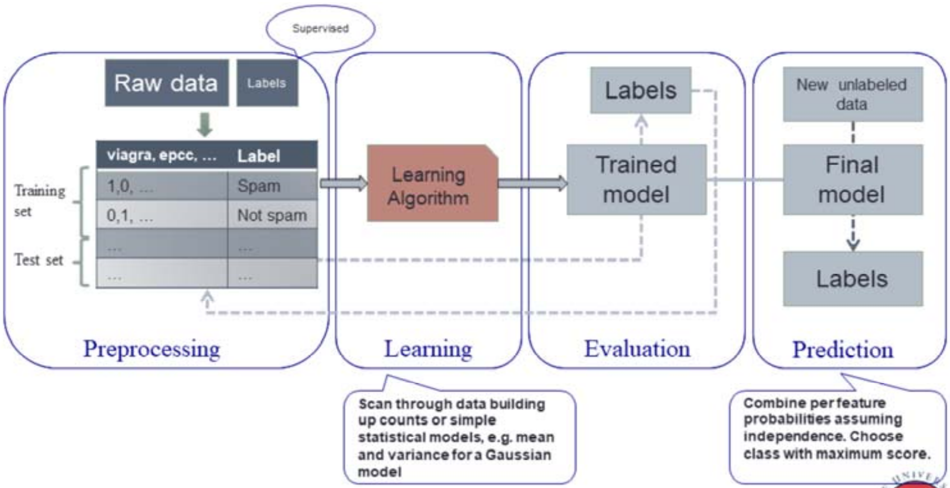
Supervised Machine Learning Classification



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Naïve Bayes implementation details

- Discrete values:
  - Learning is just a matter of counting so can be done in a single pass and easy to do in parallel
- Continuous values:
  - Single pass algorithm to compute mean and variance
  - Parallel algorithms to compute mean and variance over distributed datasets
- Map/Reduce
  - Discrete: classic counting problem
    - Map: Key=<class>:<featureName>:<featureValue> Value=1
    - Reduce: standard count reducer and combiner
  - Continuous:
    - Map: Key=<class>:<featureName> Value=<featureValue>
    - Reduce: single pass compute of mean and variance

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Applying the model

- Classification stage quick
- Can use logs to convert multiplications into additions
  - Very useful if classifying a large number of items

$$p(\mathrm{NonSpam}|\{\mathrm{viagra}=1, \mathrm{epcc}=0, \mathrm{Bayes}=0\})$$

$$\propto p(\mathrm{NonSpam})p(\mathrm{viagra}|\mathrm{NonSpam})(1-p(\mathrm{epcc}|\mathrm{NonSpam}))(1-p(\mathrm{Bayes}|\mathrm{NonSpam}))$$

$$= \frac{800}{500+800} \cdot \frac{10}{800} \cdot (1-\frac{300}{800}) \cdot (1-\frac{56}{800}) = 0.0045$$

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Naïve Bayes classification: summary

- Supervised classification
- Model
  - Fairly small, a few numbers for each feature and class combination
- Learning
  - Basic counting to build up model
  - Can often be done in a single pass, so scales well
  - Easily parallelisable
- Naïve assumption does not cause too much harm in practice
- Good first approach to get a base-line performance
- Easy to adjust weighing to get desired balance between sensitivity and specificity

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